Year 11 Mathematics Specialist

Investigation 2

***Instructions:*** *In-class to be completed under test conditions.*

*Calculators are permitted.*

Time Allocation: 40 minutes Total Marks: 30 marks

**The Birthday Problem**

### [2 marks]

Using the pigeon-hole principle prove that, in a group of 13 people, there are at least two people born in the same month of the year.

What happens in smaller groups?

Can you calculate the chance of matching birth-months?

To determine the chance of there being at least two people born in the same month in groups of less than 13 people, make the following assumptions.

* A person has an equal chance of being born in any month.  
  (This will simplify the calculations without changing the result significantly.)
* The people in the group have been randomly selected.

### [2 marks]

In a group of 2 people, what is the probability that they were both born in the same month? Give your answer as a fraction and as a percentage.

For 3 people things become a little more complicated. There is the chance of the first and second person sharing a birth-month; the first and third person might have been born in the same month or indeed so could the second and third person. There is also the possibility that all three of them had the same birth-month.

Things are getting complicated fast. Four or five people would be even more complex.

To solve this birthday problem more easily, we can work out the probability that **NO** two people will have the same birthday. If we subtract this from 1 we would have the answer to the probability of at least two sharing the same birth-month.

P(E) = 1 – P(Not E)

### [4 marks]

In a group of 3 people, the probability of at least two people being born in the same month is:

(a) Explain how this calculation was obtained.

(b) As a percentage what is the probability that, in a group of 3 people, at least two people were born in the same month?

### [3 marks]

In a group of 4 people, what is the probability that at least two people were born in the same month?

Show all working and give your final answer as a percentage.

### [3 marks]

How large must the group be, to make the probability of finding at least two people with the same birth-month is more than 90%.

Show all working.

The techniques used in the above questions can also be used to calculate the probability that two people have the same birthday. To calculate these probabilities make the following assumptions.

* Do not consider leap years. There is no 29 February and all years will be 365 days long,
* Same birthday means being born on the same date but not necessarily the same year.
* A person has an equal chance of being born on any day of the year.
* The people in all groups have been randomly selected.

### [2 marks]

Why is it important to assume all people in the group have been randomly selected?

### [4 marks]

(a) In a group of 2 people, what is the probability that they both have the same birthday?

(b) In a group of 5 people, what is the probability that at least two people have the same birthday?

### [6 marks]

*In a group of just 23 people there’s about a 50-50 chance of two people having the same birthday*.

To prove this statement is correct we will need to find a formula for solving the birthday problem.

(a) Write out the calculations for finding the probability that at least two people, in a group of 23, have the same birthday.

(You do not need to write every number. Use … within any patterns.)

(b) Within this calculation there should be examples of factorials.

Using permutation or factorial notation, simplify your above calculation.

(c) Use this formula to show that in a group of 23 people there’s about a 50-50 chance of at least two people having the same birthday.

### [4 marks]

How large must the group be to make the probability of finding two people with the same birthday at least 90%?

Show all working.

**The Birthday Problem SOLUTIONS**

### [2 marks]

The pigeon-hole principle states that if *n* items are put into *m* containers, with *n* > *m*, then at least one container must contain more than one item.

There are 13 people, , with 12 months, . Since , at least two people must have been born in the same month.

### [2 marks]

In a group of 2:

### [4 marks]

(a)

The first person can have any birth-month, .

The second person can have must have a different month to the first person, .

The third person can have must have a different month to both the first person and the second person, .

Thus the probability of three different birth months is and the probability of at least two the same is .

(b)

### [3 marks]

In a group of 4:

### [3 marks]

Two people with the same birthday more than 90%:

Therefore you need a group of 8 people.

### [2 marks]

It is important to assume all groups have been randomly selected to ensure that the rules of probability can be applied. For example a set of twins may be in a family group or in a class group. In such cases the chance of the same birthday is certain. Probability relies on all outcomes being equally likely. Random selection is required for this to be obtained.

### [4 marks]

(a) In a group of 2:

(b) In a group of 5:

### [6 marks]

(a) In a group of 23:

(b) Using permutation or factorial notation,

or

(c) Thus in a group of 23:

There’s about a 50-50 chance of two people having the same birthday.

### [4 marks]

Two people with the same birthday more than 90%:

, 89%

, 90.3%

Therefore you need a group of 41 people